

# Recovery risk measures - Developments to better protect insurance beneficiaries

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# Introduction

## Introduction

- This is a report on some research with Cosimo Munari and Stefan Weber:
- Capital Requirements and Claims Recovery: A New Perspective on Solvency Regulation, [SSRN 382 9179](#)
- We are grateful to Kerstin Awiszus and Pablo Koch for helpful discussions and comments
  
- It addresses the question

How should solvency tests, i.e. capital requirements, be designed such that they control the recovery on creditors' claims in the case of default?

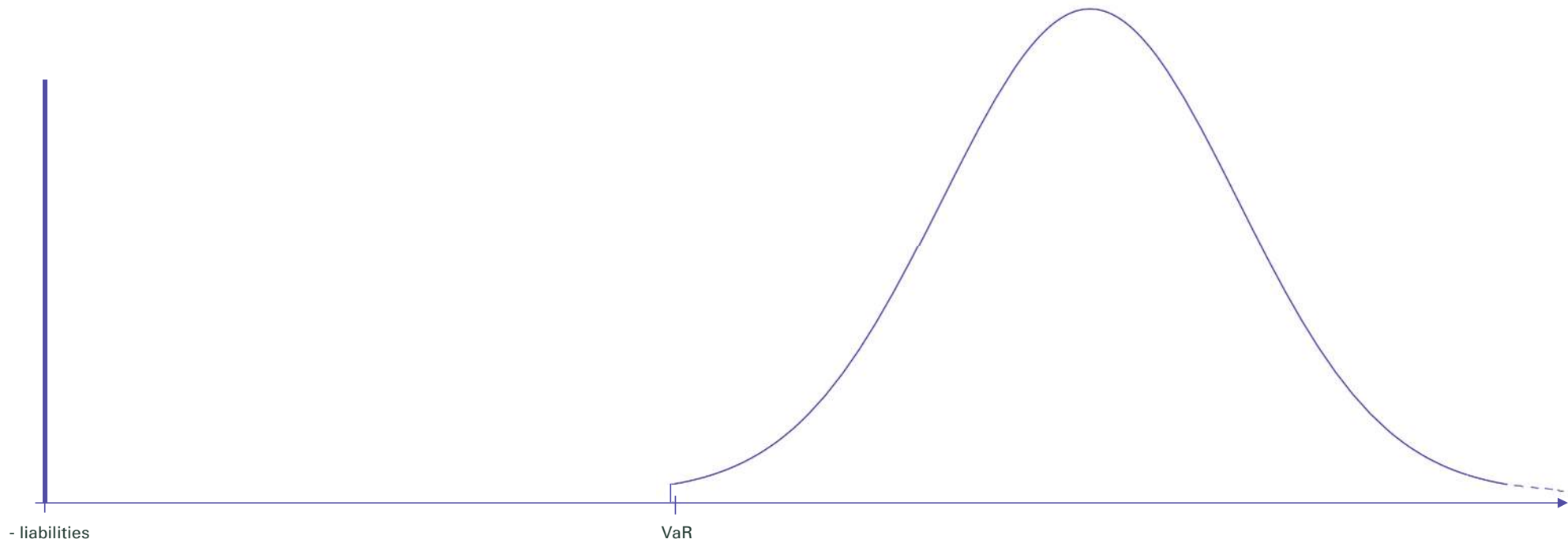
# Problem Statement



## Narrative

- Capital requirements are set for financial institutions to ensure that creditors' and insurance beneficiaries' claims are met to an *appropriate extent*.
- Current implementations of "appropriate extent": At the core of capital requirements are risk measures that ensure that creditors and insurance beneficiaries
  - incur no loss with a high probability – Value at Risk (VaR)
  - incur no loss on average over the , say, 1% worst cases – Expected Short (ES) or Average VaR (AVaR)
- However, VaR does not provide any control of the size of the loss if a loss occurs, i.e. the institution's assets can be entirely wiped out.
- For ES, assets can be annihilated in, say, half of the worst cases.
- In these cases beneficiaries do not get anything, i.e. their recovery on the claim would be nil.
- These examples are extreme – but more realistic examples can be constructed with some more effort along the same lines.
- In this sense VaR and ES fail to control, i.e. give decent lower bounds, for the recovery on claims.

## Example of a net asset value distribution with significant total loss potential and arbitrary VaR



## Appropriateness considerations

- We can ask, if creditor and insurance beneficiaries are appropriately informed, if the published solvency ratio gives no effective information on the potential loss given default they would incur:
  - there is a major difference if pensioners get a 10% reduced pension or if they do not get anything at all
- Moreover, for the prospects of restructuring the financial institution in distress the loss given default levels of the various creditors are of utmost importance.
- Finally due to knock-on effects, this might influence the stability of the financial sector.
  
- Currently, ensuring decent recovery on claims of creditors and insurance beneficiaries is mainly addressed through qualitative measures, and to a lesser extent through quantitative measures, like investment rules.
- But capital requirements play no effective role.
  
- This work tries to address this.



## Problem statement

**How should solvency tests, i.e. capital requirements, be designed such that they control the recovery on creditors' claims in the case of default of the financial institution?**

- *control*: give decent lower bounds in a suitable, e.g., probabilistic sense.
  - *default*: assets are smaller than liabilities
- Good news: it can be done by deriving Recovery Risk Measures from classical risk measures like VaR and ES, without loosing their essential properties.

# Results and Features



## Results

- I. Classical monetary risk measures are unable to provide decent lower bounds for the recovery of creditors and insurance beneficiaries in case of default.
  - Assets and liabilities must be considered separately – not just the difference.
- II. A novel risk measure, Recovery Value at Risk (RecVaR), can be defined to address this issue.
  - RecVaR can be operationally used to define a capital requirement
  - It seems to be better suited for internal and external risk management
  - It can be used to quantify shortcomings of VaR in specific situations in terms of controlling recovery
- III. The approach to derive RecVaR from VaR is flexible and allows e.g. defining Recovery Expected Shortfall (RecES) while maintaining desirable properties, e.g. subadditivity and convexity
  - Subadditivity is a prerequisite to consistently deal with diversification and decentralise risk management
  - Convexity is useful in the context of portfolio optimisation

## More Results – in the paper, but not in this presentation

- IV. The paper gives examples that illustrate how recovery risk measures react to features of the joint distribution of assets and liabilities
  - More specifically reaction to changes in marginal distribution and dependency are considered
- V. Moreover, it discusses how an appropriate calibration of a specific recovery risk measure can be done in line with the calibration of current risk measure.
- VI. The application of recovery risk measure in the context of performance management of business units is discussed.
- VII. Finally the paper shows how efficient frontiers can be computed in this case.

# Initial discussion

# Some Concepts and Ideas

## Notions

We start with a stylised balance sheet of the financial institution at time  $t$

Assets	Liabilities
$A_t$	$L_t$
	$E_t = A_t - L_t$

The standard risk measures are *Value at Risk* (V@R) and *Average Value at Risk* (AV@R) at some pre-specified level  $\alpha \in (0, 1)$ :

$$\text{V@R}_\alpha(X) := \inf\{x \in \mathbb{R}; \mathbb{P}(X + x < 0) \leq \alpha\}, \quad \text{AV@R}_\alpha(X) := \frac{1}{\alpha} \int_0^\alpha \text{V@R}_\beta(X) d\beta$$

In a VaR setting, it is the minimal amount that needs to be injected to make the situation acceptable.

The solvency test can be formulated as

$$\text{V@R}_\alpha(E_1) \leq 0 \iff \mathbb{P}(E_1 < 0) \leq \alpha \iff \mathbb{P}(E_1 \geq 0) \geq 1 - \alpha.$$

## Introducing RecVaR

Creditors receive at least a recovery fraction  $\lambda \in [0, 1]$  on their claims payments if<sup>7</sup>

$$A_1 \geq \lambda L_1 \iff E_1 + (1 - \lambda)L_1 \geq 0. \quad (5)$$

In this event, assets may not be sufficient to meet all obligations, but they cover at least a fraction  $\lambda$  of liabilities. We control recovery by imposing lower bounds on the recovery probabilities

$$\mathbb{P}(A_1 \geq \lambda L_1)$$

**Definition 3.** We denote by  $L^0$  the vector space of all random variables on some probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . Let  $\gamma : [0, 1] \rightarrow (0, 1)$  be an increasing function. The Recovery Value at Risk

$$\text{RecV@R}_\gamma : L^0 \times L^0 \rightarrow \mathbb{R} \cup \{\infty\}$$

with level function  $\gamma$  is defined by

$$\text{RecV@R}_\gamma(X, Y) := \sup_{\lambda \in [0, 1]} \text{V@R}_{\gamma(\lambda)}(X + (1 - \lambda)Y). \quad (6)$$



## RecVaR controls recovery

If the random variables  $X$  and  $Y$  in Definition 3 are interpreted, respectively, as the net asset value  $E_1$  and liabilities  $L_1$  in a company's balance sheet,<sup>9</sup> the risk measure RecV@R can be used to formulate a solvency test of the form (1). As shown in Remark 20, the condition

$$\text{RecV@R}_\gamma(\Delta E_1, L_1) \leq E_0 \quad (7)$$

is equivalent to requiring that the recovery probabilities satisfy

$$\mathbb{P}(A_1 < \lambda L_1) \leq \gamma(\lambda) \iff \mathbb{P}(A_1 \geq \lambda L_1) \geq 1 - \gamma(\lambda) \quad (8)$$

for all recovery fractions  $\lambda \in [0, 1]$ . This guarantees the desired control on the loss given default.

## Understanding RecVaR

We consider step-wise recovery functions of the form

$$\gamma(\lambda) = \begin{cases} \alpha_1 & \text{if } 0 = r_0 \leq \lambda < r_1, \\ \alpha_2 & \text{if } r_1 \leq \lambda < r_2, \\ \vdots & \\ \alpha_n & \text{if } r_{n-1} \leq \lambda < r_n, \\ \alpha_{n+1} & \text{if } r_n \leq \lambda \leq r_{n+1} = 1, \end{cases} \quad (10)$$

with  $0 < \alpha_1 < \dots < \alpha_{n+1} < 1$  and  $0 < r_1 < \dots < r_n < 1$ . The parameters  $r_i$  correspond to critical target recovery fractions while the parameters  $\alpha_i$  define bounds on the corresponding recovery probabilities for every  $i = 1, \dots, n+1$ . As shown in the next proposition, the RecV@R induced by such recovery functions can be expressed as a maximum of finitely many V@R's.

**Proposition 5.** *Let  $\gamma$  be defined as in (10). Then, for all  $X, Y \in L^0$  with  $Y \geq 0$*

$$\text{RecV@R}_\gamma(X, Y) = \max_{i=1, \dots, n+1} \text{V@R}_{\alpha_i}(X + (1 - r_i)Y).$$

## Features of RecVaR

**Proposition 6.** *The risk measure  $\text{RecV@R}_\gamma$  has the following properties:*

(a) Cash invariance in the first component: *For all  $X, Y \in L^0$  and  $m \in \mathbb{R}$*

$$\text{RecV@R}_\gamma(X + m, Y) = \text{RecV@R}_\gamma(X, Y) - m.$$

(b) Monotonicity: *For all  $X_1, X_2, Y_1, Y_2 \in L^0$  with  $X_1 \geq X_2$  and  $Y_1 \geq Y_2$   $\mathbb{P}$ -almost surely<sup>12</sup>*

$$\text{RecV@R}_\gamma(X_1, Y_1) \leq \text{RecV@R}_\gamma(X_2, Y_2).$$

(c) Positive homogeneity: *For all  $X, Y \in L^0$  and  $a \in [0, \infty)$*

$$\text{RecV@R}_\gamma(aX, aY) = a\text{RecV@R}_\gamma(X, Y).$$

(d) Star-shapedness<sup>13</sup> in the first component: *For all  $X, Y \in L^0$  with  $Y \geq 0$  and  $a \in [1, \infty)$*

$$\text{RecV@R}_\gamma(aX, Y) \geq a\text{RecV@R}_\gamma(X, Y).$$

(e) Normalization: *For every  $Y \in L^0$  with  $Y \geq 0$  we have  $\text{RecV@R}_\gamma(0, Y) = 0$ .*

(f) Finiteness: *For all  $X, Y \in L^0$  with  $Y \geq 0$  we have  $\text{RecV@R}_\gamma(X, Y) < \infty$  under any of the following conditions:  $\gamma(0) > 0$ ,  $\text{V@R}_{\gamma(0)}(X) < \infty$ , or  $X$  is bounded from below.*

## RecVaR's interpretation as a capital requirement – adding/extracting assets

The previous proposition shows that  $\text{RecV@R}$  is a standard monetary risk measure in its first component and can conveniently be expressed as a capital requirement:

$$\begin{aligned}\text{RecV@R}_\gamma(E_1, L_1) &= \inf\{m \in \mathbb{R}; \text{RecV@R}_\gamma(E_1 + m, L_1) \leq 0\} \\ &= \inf\{m \in \mathbb{R}; \mathbb{P}(A_1 + m < \lambda L_1) \leq \gamma(\lambda), \forall \lambda \in [0, 1]\} \\ &= \inf\{m \in \mathbb{R}; \mathbb{P}(A_1 + m \geq \lambda L_1) \geq 1 - \gamma(\lambda), \forall \lambda \in [0, 1]\},\end{aligned}$$

where the second equality is a consequence of Remark [20](#). This leads to the following useful operational interpretation of  $\text{RecV@R}$ :

- If  $\text{RecV@R}_\gamma(E_1, L_1) > 0$ , the company fails to pass the recovery-based solvency test [\(7\)](#) and  $\text{RecV@R}_\gamma(E_1, L_1)$  is the minimal amount of cash that needs to be added to its assets in order to become adequately capitalized.
- If  $\text{RecV@R}_\gamma(E_1, L_1) < 0$ , the company is adequately capitalized according to the recovery-based solvency test [\(7\)](#) and  $-\text{RecV@R}_\gamma(E_1, L_1)$  is the maximal amount of cash that may be extracted from the asset side without compromising capital adequacy.

## Forcing cash invariance in the second component ...

*This leads to the following operational interpretation:*

- *If  $\text{LRecV@R}_\gamma(A_1, L_1) > 0$ , the company fails the solvency test (13) and  $\text{LRecV@R}_\gamma(A_1, L_1)$  is the minimal nominal amount of liabilities that needs to be removed from the balance sheet in order to pass the test, e.g., by transferring these liabilities to suitable equity holders outside the firm.*
- *If  $\text{LRecV@R}_\gamma(A_1, L_1) < 0$ , the company is adequately capitalized. The company may at most create an additional amount  $-\text{LRecV@R}_\gamma(A_1, L_1)$  of liabilities, e.g., via additional debt, and immediately distribute the same amount of cash to its shareholders.*

*Observe that assets  $A_1$  and liabilities  $L_1$  are used in the definition of  $\text{LRecV@R}$  instead of the net asset value  $E_1$  and liabilities  $L_1$  in order to obtain a simple cash-invariant recovery risk measure with a transparent operational interpretation<sup>14</sup> which is still equivalent to condition (8). Note that  $\text{LRecV@R}$  is cash invariant (in the appropriate sense) with respect to its second argument, i.e., for all  $A_1, L_1 \in L_+^0$  and  $m \in \mathbb{R}$*

$$\text{LRecV@R}_\gamma(A_1, L_1 + m) = \text{LRecV@R}_\gamma(A_1, L_1) + m.$$



## ...LRecVaR's interpretation as a capital requirement – adding/extracting liabilities

*This leads to the following operational interpretation:*

- *If  $\text{LRecV@R}_\gamma(A_1, L_1) > 0$ , the company fails the solvency test (13) and  $\text{LRecV@R}_\gamma(A_1, L_1)$  is the minimal nominal amount of liabilities that needs to be removed from the balance sheet in order to pass the test, e.g., by transferring these liabilities to suitable equity holders outside the firm.*
- *If  $\text{LRecV@R}_\gamma(A_1, L_1) < 0$ , the company is adequately capitalized. The company may at most create an additional amount  $-\text{LRecV@R}_\gamma(A_1, L_1)$  of liabilities, e.g., via additional debt, and immediately distribute the same amount of cash to its shareholders.*

*Observe that assets  $A_1$  and liabilities  $L_1$  are used in the definition of  $\text{LRecV@R}$  instead of the net asset value  $E_1$  and liabilities  $L_1$  in order to obtain a simple cash-invariant recovery risk measure with a transparent operational interpretation.<sup>14</sup>*

- In portfolio transfers, e.g. in restructuring, we will need to consider both RecVaR and LRecVaR as assets and liabilities get transferred.
- However, for recapitalisation, RecVaR remains decisive.

## General recovery risk measure

**Definition 8.** Let  $L^0$  be the set of random variables on some probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . We denote by  $\mathcal{X} \subseteq L^0$  a vector space that contains the constants. For every  $\lambda \in [0, 1]$  consider a map  $\rho_\lambda : \mathcal{X} \rightarrow \mathbb{R} \cup \{\infty\}$  and assume that  $\rho_{\lambda_1} \geq \rho_{\lambda_2}$  whenever  $\lambda_1 \leq \lambda_2$ . The recovery risk measure

$$\text{Rec}\rho : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R} \cup \{\infty\}$$

is defined by

$$\text{Rec}\rho(X, Y) := \sup_{\lambda \in [0, 1]} \rho_\lambda(X + (1 - \lambda)Y). \quad (14)$$

In line with our discussion on  $\text{RecV@R}$ , if the random variables  $X$  and  $Y$  in Definition 8 are respectively interpreted as the net asset value  $E_1$  and liabilities  $L_1$  in a company's balance sheet, the recovery risk measure  $\text{Rec}\rho$  can be employed to formulate a solvency test of the form (1). Indeed, similarly to what we have shown in Remark 20, we have

$$\text{Rec}\rho(\Delta E_1, L_1) \leq E_0 \iff \forall \lambda \in [0, 1] : \rho_\lambda(A_1 - \lambda L_1) \leq 0. \quad (15)$$

The specific interpretation of this recovery-based solvency test will, of course, depend on the choice of the monetary risk measures used to build  $\text{Rec}\rho$ .

## Features

- Recovery risk measure inherit all the desired properties, from the underlying family of risk measures
- Expected Shortfall gives rise to a recovery version mapping the recovery level to be archived in expectation to the respective quantiles.
- RecES inherits all the nice properties of ES.
- A liability version LRecES can be constructed as for RecVaR with an equivalent interpretation



## Consideration for introducing recovery risk measures

- Introducing a recovery measure RecVaR or RecES with a piecewise constant recovery to quantile function seems appropriate. This would increase modelling and calculation burden in a well controlled and appropriate fashion. The introduction would not increase operational risk significantly as the components to be calculated remain similar.
- A recalibration should be considered to ensure that e.g. the total capital requirement for a market remains constant. Companies with benign recovery in the tail would benefit from a lower capital requirement.
- Cost allocation for performance attribution should become even more stable, esp. in the case of VaR.
- The impact on asset-liability management should be further analysed. It might lead to incentives to manage deviations even more closely.
- Introducing recovery measures to regulatory systems should involve testing by volunteers and some field testing.
- It should be analysed which of the supervisory measures that currently aim to ensure decent recovery become obsolete

# Discussion

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